

BCA -105(N)

Roll No.

B.C.A. (First Semester)
EXAMINATION, Dec.-2015
(New Course)

Paper – Fifth

MATHEMATICS-I

[Time: Three Hours]

[Maximum Marks: 75]

Note : Attempt questions from all the questions as directed.

SECTION – A

Note : Attempt all the questions, each will carry 3 marks.

Short Answer Type Questions Only

1. A Show that the adjoint of

$$\begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

is three times its transpose.

- B. Find the equation of the line joining (1, 2) and (3, 6) using determinants.

C. If $f(x) = \begin{cases} \frac{\sin [x]}{[x]}, & \text{if } [x] \neq 0 \\ 0, & \text{if } [x] = 0 \end{cases}$

Where $[x]$ denotes the greatest integer less than or equal to x , then find $\lim_{x \rightarrow 0} f(x)$.

- D. Show that $f(x) = x|x|$ is differentiable at $x=0$.
- E. Differentiate $\tan^{-1} \frac{x}{\sqrt{1-x^2}}$ with respect to $\cos^{-1}(2x^2 - 1)$.
- F. Evaluate $\int_0^{\infty} \frac{x^8(1-x)^6}{(1+x)^{24}} dx$
- G. Evaluate $\int \frac{dx}{x(x^n+1)}$
- H. Find the angle between the vectors $\vec{a} = 2\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = 6\hat{i} - 3\hat{j} + 2\hat{k}$.
- I. If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 5\hat{i} + 5\hat{j} - \hat{k}$, find $(\vec{a} + \vec{b}) \times 2\vec{b}$ and $(\vec{a} + \vec{b}) \times 2\vec{b}$

SECTION - B

12 Marks Each Question

Long Answer Type Questions

(02 out of 04 questions)

Find the Eigen value and Eigen vector of the following matrix

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$$

Expand $e^{\sin^{-1}x}$ by Maclaurin's theorem and find the general term.

4. (a) Reduce the following matrix into normal form and hence find its rank

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

- (b) Prove that

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$$

5. (a) Show that the following limit does not exist

$$\lim_{x \rightarrow 3} \frac{|x-3|}{x-3}$$
- (b) Discuss the nature of discontinuity of the following function at the origin

$$f(x) = \begin{cases} \frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}$$

SECTION - C

12 Marks Each Question

Long Answer Type Questions

(02 out of 04 questions)

6. (a) Discuss the maxima and minima of the function

$$f(x) = \frac{x}{1+x^2}$$

- (b) Trace the curve

$$ay^2 = x^2(a-x)$$

7. (a) Evaluate $\int \sec^3 x \, dx$

- (b) if $\ln = \int_0^{\pi/4} \tan^n x \, dx$,

$$\text{show that } \ln + \ln - 2 = \frac{1}{n-1}$$

8. (a) Evaluate $\int_0^{\infty} x^m e^{-ax^n} \, dx$, when m, n, a are positive constants.

- (b) Evaluate $\int \cos x \cos 2x \cos 3x \, dx$

9. (a) If $(\vec{a}, \vec{b}, \vec{c})$ are three non-coplanar vectors show that

$$[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2$$

- (b) Prove that

$$\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}$$